from II, $t=a_4/u=\frac{25}{5}=5$,

from (7), $d=\pm\sqrt{5-1}=\pm 2$,

from (8), $b=\pm\sqrt{5-4}=\pm 1$.

The roots are therefore $2\pm i$, $1\pm 2i$.

The value 2 for a satisfies the equation III as it should, and 1 and 2 are the only real roots which III possesses.

If the given equation (B) has two real roots and both are known to any desired degree of accuracy, the two other roots are very easily found. Put

$$a+bi=h$$
$$a-bi=k$$

where h and k are known. Then

$$a=\tfrac{1}{2}(h+k),$$
$$b=-\tfrac{1}{2}(h-k)i,$$
$$c=-\tfrac{1}{2}(h+k+a_1) \text{ from (5)},$$

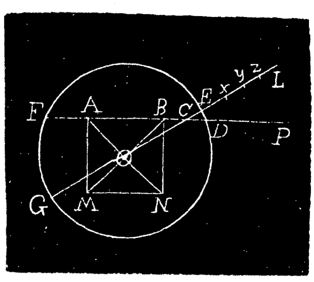
and u is found from I and d from (7) as before. Thus the roots are all determined.

If all of the roots of (B) are real, they will be equally well given by the first method above. In this case b and d will be imaginary.

A DEVICE FOR EXTRACTING THE SQUARE ROOT OF CERTAIN SURD QUANTITIES.

By ROBERT J. ALEY, A. M., Ph. D., Professor of Mathematics, University of Indiana, Bloomington, Indiana.

$ABMN$ is a square. OL is an arm revolving freely about O. This arm beyond C is divided into equal parts at E, x, y, z, etc.



To determine the character of the divisions made on FP by the points of division on OL as OL revolves. Call the side of the square AB, $2a$; BC, b; CE, c; and CD, x.

Then $OC=\sqrt{a^2+(a+b)^2}$.

$GC=2\sqrt{a^2+(a+b)^2}+c$.

$$FC=2(a+b)+x.$$

From the properties of two intersecting chords we have,

$$x\{x+2(a+b)\}=c\{c+2\sqrt{a^2+(a+b)^2}\}$$

$$x^2+2(a+c)x+(a+b)^2=a^2+2ab+b^2+c^2+2c\sqrt{a^2+(a+b)^2}$$

$$x+(a+b)=\sqrt{(a+b)^2+c^2+2c\sqrt{a^2+(a+b)^2}.}$$

Suppose that we examine the results when integral values are given to the constants.

Put $a=c=1$, $b=0$. (Let c take successively the values 1, 2, 3, 4, ... etc.)

Then $x+1=\sqrt{2+2\sqrt{2}},$

$$x+1=\sqrt{5+4\sqrt{2}},$$

$$x+1=\sqrt{10+6\sqrt{2},}$$

$$x+1=\sqrt{17+8\sqrt{2},}\text{ etc.},$$

and the law of the series is readily seen.

Put $a=c=b=1$, and let c vary as before.

$$x+2=\sqrt{5+2\sqrt{5}},$$

$$x+2=\sqrt{8+4\sqrt{5}},$$

$$x+2=\sqrt{13+6\sqrt{5},}$$

$$x+2=\sqrt{20+8\sqrt{5},}\text{ etc.}$$

The law is again evident.

Put $a=1$, $b=2$, and let c vary.

$$x+3=\sqrt{10+2\sqrt{10}},$$

$$x+3=\sqrt{13+4\sqrt{10}},$$

$$x+3=\sqrt{18+6\sqrt{10}},$$

$$x+3=\sqrt{25+8\sqrt{10}},\text{ etc.}$$

The law is again evident.

Put $a=1$, $b=3$, and let c vary.

$$x+4=\sqrt{17+2\sqrt{17}},$$

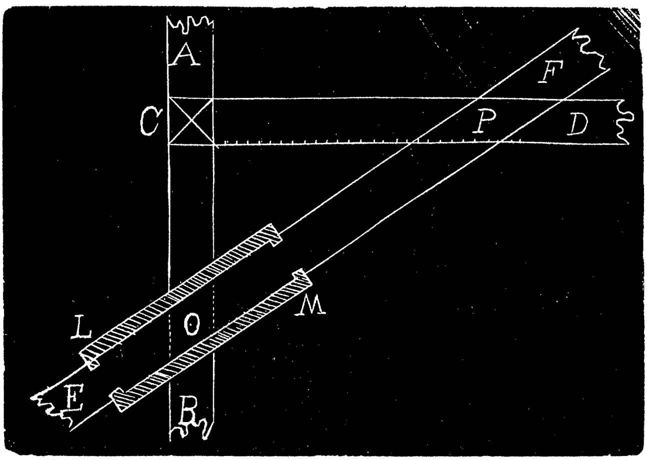
$$x+4=\sqrt{20+4\sqrt{17}},$$

$$x+4=\sqrt{25+6\sqrt{17}}.$$

These examples show how the various series will be found.

From the previous considerations we at once have the data for the construction of a simple mechanical device for the extraction of roots of certain surd quantities.

AB is an upright so arranged that CD will slide up and down always parallel to itself. It is accurately marked to scale so that CD may be set at any de-



sired a. FE works in a slide LM which is free to rotate about O. It is accurately ruled to scale from P to F. By sliding it in LM, P may be set at any desired $a+c$. CD is ruled to scale and is also provided with a diagonal scale, so that by the use of dividers, results may be read to hundredths. When the instrument is set at any chosen a and b, all the roots for that set may be read off at once.

Tables may be easily constructed. A few samples are here given.

The a's are read in the vertical columns, the b's horizontally, and in the squares the c's take successively the values 1, 2, 3, etc. But three terms are given in each square, enough to make the law perfectly evident.

TABLE FOR INTEGRAL VALUES OF a, b, AND c.

$a=$ \ $b=$	0	1	2	3	4	5	6	7	8
1	$\sqrt{2+2\sqrt{2}}$ $\sqrt{5+4\sqrt{2}}$ $\sqrt{10+6\sqrt{2}}$	$\sqrt{5+2\sqrt{5}}$ $\sqrt{8+4\sqrt{5}}$ $\sqrt{13+6\sqrt{5}}$	$\sqrt{10+2\sqrt{10}}$ $\sqrt{13+4\sqrt{10}}$ $\sqrt{18+6\sqrt{10}}$	$\sqrt{17+2\sqrt{17}}$ $\sqrt{20+4\sqrt{17}}$ $\sqrt{25+6\sqrt{17}}$	$\sqrt{26+2\sqrt{26}}$ $\sqrt{29+4\sqrt{26}}$ $\sqrt{34+6\sqrt{26}}$	$\sqrt{37+2\sqrt{37}}$ $\sqrt{40+4\sqrt{37}}$ $\sqrt{45+6\sqrt{37}}$	$\sqrt{50+2\sqrt{50}}$ $\sqrt{53+4\sqrt{50}}$ $\sqrt{58+6\sqrt{50}}$	$\sqrt{65+2\sqrt{65}}$ $\sqrt{68+4\sqrt{65}}$ $\sqrt{73+6\sqrt{65}}$	$\sqrt{82+2\sqrt{82}}$ $\sqrt{85+4\sqrt{82}}$ $\sqrt{90+6\sqrt{82}}$
2	$\sqrt{5+2\sqrt{8}}$ $\sqrt{8+4\sqrt{8}}$ $\sqrt{13+6\sqrt{8}}$	$\sqrt{10+2\sqrt{13}}$ $\sqrt{13+4\sqrt{13}}$ $\sqrt{18+6\sqrt{13}}$	$\sqrt{17+2\sqrt{20}}$ $\sqrt{20+4\sqrt{20}}$ $\sqrt{25+6\sqrt{20}}$	$\sqrt{26+2\sqrt{29}}$ $\sqrt{29+4\sqrt{29}}$ $\sqrt{34+6\sqrt{29}}$	$\sqrt{37+2\sqrt{40}}$ $\sqrt{40+4\sqrt{40}}$ $\sqrt{45+6\sqrt{40}}$	$\sqrt{50+2\sqrt{53}}$ $\sqrt{53+4\sqrt{53}}$ $\sqrt{58+6\sqrt{53}}$	$\sqrt{65+2\sqrt{68}}$ $\sqrt{68+4\sqrt{68}}$ $\sqrt{73+6\sqrt{68}}$	$\sqrt{82+2\sqrt{85}}$ $\sqrt{85+4\sqrt{85}}$ $\sqrt{90+6\sqrt{85}}$	
3	$\sqrt{10+2\sqrt{18}}$ $\sqrt{13+4\sqrt{18}}$ $\sqrt{18+6\sqrt{18}}$	$\sqrt{17+2\sqrt{25}}$ $\sqrt{20+4\sqrt{25}}$ $\sqrt{25+6\sqrt{25}}$	$\sqrt{26+2\sqrt{34}}$ $\sqrt{29+4\sqrt{34}}$ $\sqrt{34+6\sqrt{34}}$	$\sqrt{37+2\sqrt{45}}$ $\sqrt{40+4\sqrt{45}}$ $\sqrt{45+6\sqrt{45}}$	$\sqrt{50+2\sqrt{58}}$ $\sqrt{53+4\sqrt{58}}$ $\sqrt{58+6\sqrt{58}}$	$\sqrt{65+2\sqrt{73}}$ $\sqrt{68+4\sqrt{73}}$ $\sqrt{73+6\sqrt{73}}$	$\sqrt{82+2\sqrt{90}}$ $\sqrt{85+4\sqrt{90}}$ $\sqrt{90+6\sqrt{90}}$		
4	$\sqrt{17+2\sqrt{32}}$ $\sqrt{20+4\sqrt{32}}$ $\sqrt{25+6\sqrt{32}}$	$\sqrt{26+2\sqrt{41}}$ $\sqrt{29+4\sqrt{41}}$ $\sqrt{34+6\sqrt{41}}$	$\sqrt{37+2\sqrt{52}}$ $\sqrt{40+4\sqrt{52}}$ $\sqrt{45+6\sqrt{52}}$	$\sqrt{50+2\sqrt{65}}$ $\sqrt{53+4\sqrt{65}}$ $\sqrt{58+6\sqrt{65}}$	$\sqrt{65+2\sqrt{80}}$ $\sqrt{68+4\sqrt{80}}$ $\sqrt{73+6\sqrt{80}}$	$\sqrt{82+2\sqrt{97}}$ $\sqrt{85+4\sqrt{97}}$ $\sqrt{90+6\sqrt{97}}$			
5	$\sqrt{26+2\sqrt{50}}$ $\sqrt{29+4\sqrt{50}}$ $\sqrt{34+6\sqrt{50}}$	$\sqrt{37+2\sqrt{61}}$ $\sqrt{40+4\sqrt{61}}$ $\sqrt{45+6\sqrt{61}}$	$\sqrt{50+2\sqrt{74}}$ $\sqrt{53+4\sqrt{74}}$ $\sqrt{58+6\sqrt{74}}$	$\sqrt{65+2\sqrt{89}}$ $\sqrt{68+4\sqrt{89}}$ $\sqrt{73+6\sqrt{89}}$	$\sqrt{82+2\sqrt{106}}$ $\sqrt{85+4\sqrt{106}}$ $\sqrt{90+6\sqrt{106}}$				
6	$\sqrt{37+2\sqrt{72}}$ $\sqrt{40+4\sqrt{72}}$ $\sqrt{45+6\sqrt{72}}$	$\sqrt{50+2\sqrt{85}}$ $\sqrt{53+4\sqrt{85}}$ $\sqrt{58+6\sqrt{85}}$	$\sqrt{65+2\sqrt{100}}$ $\sqrt{68+4\sqrt{100}}$ $\sqrt{73+6\sqrt{100}}$	$\sqrt{82+2\sqrt{117}}$ $\sqrt{85+4\sqrt{117}}$ $\sqrt{90+6\sqrt{117}}$					
7	$\sqrt{50+2\sqrt{98}}$ $\sqrt{53+4\sqrt{98}}$ $\sqrt{58+6\sqrt{98}}$	$\sqrt{65+2\sqrt{113}}$ $\sqrt{68+4\sqrt{113}}$ $\sqrt{73+6\sqrt{113}}$	$\sqrt{82+2\sqrt{130}}$ $\sqrt{85+4\sqrt{130}}$ $\sqrt{90+6\sqrt{130}}$						
8	$\sqrt{65+2\sqrt{128}}$ $\sqrt{68+4\sqrt{128}}$ $\sqrt{73+6\sqrt{128}}$	$\sqrt{82+2\sqrt{145}}$ $\sqrt{85+4\sqrt{145}}$ $\sqrt{90+6\sqrt{145}}$							
9	$\sqrt{82+2\sqrt{162}}$ $\sqrt{85+4\sqrt{162}}$ $\sqrt{90+6\sqrt{162}}$								

Fractional Values of a and b.

a \ $b=$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	$\frac{9}{2}$
$a=\frac{1}{2}$	$\sqrt{2+\sqrt{5}}$ $\sqrt{5+2\sqrt{5}}$ $\sqrt{10+3\sqrt{5}}$	$\sqrt{5+\sqrt{17}}$ $\sqrt{8+2\sqrt{17}}$ $\sqrt{13+3\sqrt{17}}$	$\sqrt{10+\sqrt{37}}$ $\sqrt{13+2\sqrt{37}}$ $\sqrt{18+3\sqrt{37}}$	$\sqrt{17+\sqrt{65}}$ $\sqrt{20+2\sqrt{65}}$ $\sqrt{25+3\sqrt{65}}$	$\sqrt{26+\sqrt{101}}$ $\sqrt{29+2\sqrt{101}}$ $\sqrt{34+3\sqrt{101}}$
$\frac{3}{2}$	$\sqrt{5+\sqrt{25}}$ $\sqrt{8+2\sqrt{25}}$ $\sqrt{13+3\sqrt{25}}$	$\sqrt{10+\sqrt{45}}$ $\sqrt{13+2\sqrt{45}}$ $\sqrt{18+3\sqrt{45}}$	$\sqrt{17+\sqrt{73}}$ $\sqrt{20+2\sqrt{73}}$ $\sqrt{25+3\sqrt{73}}$	$\sqrt{26+\sqrt{109}}$ $\sqrt{29+2\sqrt{109}}$ $\sqrt{34+3\sqrt{109}}$	
$\frac{5}{2}$	$\sqrt{10+\sqrt{61}}$ $\sqrt{13+2\sqrt{61}}$ $\sqrt{18+3\sqrt{61}}$	$\sqrt{17+\sqrt{89}}$ $\sqrt{20+2\sqrt{89}}$ $\sqrt{25+3\sqrt{89}}$	$\sqrt{26+\sqrt{125}}$ $\sqrt{29+2\sqrt{125}}$ $\sqrt{34+3\sqrt{125}}$		
$\frac{7}{2}$	$\sqrt{17+\sqrt{113}}$ $\sqrt{20+2\sqrt{113}}$ $\sqrt{25+3\sqrt{113}}$	$\sqrt{26+\sqrt{149}}$ $\sqrt{29+2\sqrt{149}}$ $\sqrt{34+3\sqrt{149}}$			
$\frac{9}{2}$	$\sqrt{26+\sqrt{181}}$ $\sqrt{29+2\sqrt{181}}$ $\sqrt{34+3\sqrt{181}}$				

Simple Surd Values of a and b.

a \ $b=$	0	$\sqrt{5}$	$2\sqrt{5}$	$3\sqrt{5}$
$\sqrt{5}$	$\sqrt{6+2\sqrt{10}}$ $\sqrt{9+4\sqrt{10}}$ $\sqrt{14+6\sqrt{10}}$	$\sqrt{21+2\sqrt{25}}$ $\sqrt{24+4\sqrt{25}}$ $\sqrt{29+6\sqrt{25}}$	$\sqrt{46+2\sqrt{50}}$ $\sqrt{49+4\sqrt{50}}$ $\sqrt{54+6\sqrt{50}}$	$\sqrt{81+2\sqrt{85}}$ $\sqrt{84+4\sqrt{85}}$ $\sqrt{89+6\sqrt{85}}$
$2\sqrt{5}$	$\sqrt{21+2\sqrt{40}}$ $\sqrt{24+4\sqrt{40}}$ $\sqrt{29+6\sqrt{40}}$	$\sqrt{46+2\sqrt{65}}$ $\sqrt{49+4\sqrt{65}}$ $\sqrt{54+6\sqrt{65}}$	$\sqrt{81+2\sqrt{100}}$ $\sqrt{84+4\sqrt{100}}$ $\sqrt{89+6\sqrt{100}}$	
$3\sqrt{5}$	$\sqrt{46+2\sqrt{90}}$ $\sqrt{49+4\sqrt{90}}$ $\sqrt{54+6\sqrt{90}}$	$\sqrt{81+2\sqrt{125}}$ $\sqrt{84+4\sqrt{125}}$ $\sqrt{89+6\sqrt{125}}$		
$4\sqrt{5}$	$\sqrt{81+2\sqrt{160}}$ $\sqrt{84+4\sqrt{160}}$ $\sqrt{89+6\sqrt{160}}$			

Simple Surd Values of a and b.

a \ $b=$	0	$\sqrt{2}$	$2\sqrt{2}$	$3\sqrt{2}$
$a=\sqrt{2}$	$\sqrt{3+2\sqrt{4}}$ $\sqrt{6+4\sqrt{4}}$ $\sqrt{11+6\sqrt{4}}$	$\sqrt{9+2\sqrt{10}}$ $\sqrt{12+4\sqrt{10}}$ $\sqrt{17+6\sqrt{10}}$	$\sqrt{19+2\sqrt{20}}$ $\sqrt{22+4\sqrt{20}}$ $\sqrt{25+6\sqrt{20}}$	$\sqrt{33+2\sqrt{34}}$ $\sqrt{36+4\sqrt{34}}$ $\sqrt{41+6\sqrt{34}}$
$2\sqrt{2}$	$\sqrt{9+2\sqrt{16}}$ $\sqrt{12+4\sqrt{16}}$ $\sqrt{17+6\sqrt{16}}$	$\sqrt{19+2\sqrt{26}}$ $\sqrt{22+4\sqrt{26}}$ $\sqrt{25+6\sqrt{26}}$	$\sqrt{33+2\sqrt{40}}$ $\sqrt{36+4\sqrt{40}}$ $\sqrt{41+6\sqrt{40}}$	
$3\sqrt{2}$	$\sqrt{19+2\sqrt{36}}$ $\sqrt{22+4\sqrt{36}}$ $\sqrt{27+6\sqrt{36}}$	$\sqrt{33+2\sqrt{50}}$ $\sqrt{36+4\sqrt{50}}$ $\sqrt{41+6\sqrt{50}}$		
$4\sqrt{2}$	$\sqrt{33+2\sqrt{64}}$ $\sqrt{36+4\sqrt{64}}$ $\sqrt{41+6\sqrt{64}}$			

Simple Surd Values of a and b.

a \ $b=$	0	$\sqrt{3}$	$2\sqrt{3}$	$3\sqrt{3}$
$\sqrt{3}$	$\sqrt{4+2\sqrt{6}}$ $\sqrt{7+4\sqrt{6}}$ $\sqrt{12+6\sqrt{6}}$	$\sqrt{13+2\sqrt{15}}$ $\sqrt{16+4\sqrt{15}}$ $\sqrt{21+6\sqrt{15}}$	$\sqrt{28+2\sqrt{30}}$ $\sqrt{31+4\sqrt{30}}$ $\sqrt{36+6\sqrt{30}}$	$\sqrt{49+2\sqrt{51}}$ $\sqrt{52+4\sqrt{51}}$ $\sqrt{57+6\sqrt{51}}$
$2\sqrt{3}$	$\sqrt{13+2\sqrt{24}}$ $\sqrt{16+4\sqrt{24}}$ $\sqrt{21+6\sqrt{24}}$	$\sqrt{28+2\sqrt{39}}$ $\sqrt{31+4\sqrt{39}}$ $\sqrt{36+6\sqrt{39}}$	$\sqrt{49+2\sqrt{60}}$ $\sqrt{52+4\sqrt{60}}$ $\sqrt{57+6\sqrt{60}}$	
$3\sqrt{3}$	$\sqrt{28+2\sqrt{54}}$ $\sqrt{31+4\sqrt{54}}$ $\sqrt{36+6\sqrt{54}}$	$\sqrt{49+2\sqrt{75}}$ $\sqrt{52+4\sqrt{75}}$ $\sqrt{57+6\sqrt{75}}$		
$4\sqrt{3}$	$\sqrt{49+2\sqrt{96}}$ $\sqrt{51+4\sqrt{96}}$ $\sqrt{56+6\sqrt{96}}$			